Suppose you and some friends plan to go to a movie where tickets cost $8 each.

You will pay $8 for 1 ticket, $16 for 2 tickets, $24 for 3 tickets, $32 for 4 tickets, and so on. The ratios of the total cost of the tickets to the number of tickets are all equivalent.

A group of ratios that are equivalent are in a proportional relationship. When ratios are equivalent, they all have the same unit rate. In a proportional relationship, the unit rate is called the constant of proportionality.

The table below shows the total cost of movie tickets based on the number of tickets you buy.

<table>
<thead>
<tr>
<th>Total Cost of Tickets ($)</th>
<th>8</th>
<th>16</th>
<th>24</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Tickets</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

The ratios of the total cost of tickets to the number of tickets are equivalent. The ratios all simplify to \(\frac{8}{1} \) or 8, so the ratios are in a proportional relationship.

\[
\frac{8}{1} = 8 \quad \frac{16}{2} = 8 \quad \frac{24}{3} = 8 \quad \frac{32}{4} = 8
\]

The unit rate is 8, so the constant of proportionality is 8. The equation \(c = 8t\), where \(c\) is the total cost and \(t\) is the number of tickets, represents this relationship. The total cost is always 8 times the number of tickets.

The table below shows the cost to play in the town soccer tournament.

<table>
<thead>
<tr>
<th>Total Cost ($)</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Family Members</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

You can find and simplify the ratios of the total cost to the number of family members.

\[
\frac{7}{1} = 7 \quad \frac{8}{2} = 4 \quad \frac{9}{3} = 3 \quad \frac{10}{4} = 2 \frac{1}{2}
\]

The ratios are not equivalent, so the quantities are not in a proportional relationship.
Think  How can you use a graph to tell if a relationship is proportional?

You can use a graph to determine if a relationship is proportional.

The data for the cost of movie tickets and the cost to participate in the soccer tournament can be modeled by the graphs below.

The points on the graphs are on a straight line for both sets of data, but only the data for the cost of movie tickets goes through the origin. This means that only the total cost of the movie tickets compared to the number of tickets is a proportional relationship.

<table>
<thead>
<tr>
<th>Proportional Relationship</th>
<th>Non-Proportional Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>• The graph can be represented by a straight line.</td>
<td>• The graph may or may not be represented by a straight line.</td>
</tr>
<tr>
<td>• The line goes through the origin.</td>
<td>• If the graph is a line, it does not go through the origin.</td>
</tr>
</tbody>
</table>

Reflect  1  Look at the graph that compares the total cost to the number of movie tickets you buy. How can you identify the constant of proportionality in the graph?
Explore It

Use the table below to analyze the cost of downloading applications to a phone.

<table>
<thead>
<tr>
<th>Number of Downloads</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost ($)</td>
<td>6</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>30</td>
</tr>
</tbody>
</table>

2 How can you find the ratio of the total cost to the number of downloads?

3 What is the ratio of the total cost to the number of downloads when you download
   2 applications? _____ 4 applications? _____ 5 applications? _____
   6 applications? _____ 10 applications? _____

4 Are the data in the table in a proportional relationship? If so, what is the constant
   of proportionality?

Now try these problems.

5 The table shows the number of hours needed for different numbers of people to clean up after a school dance.

<table>
<thead>
<tr>
<th>Hours Needed to Clean Up</th>
<th>12</th>
<th>9</th>
<th>8</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of People Cleaning</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

Are the quantities in the table in a proportional relationship? Explain your reasoning.

6 The students in the Service Club are mixing paint to make a mural. The table below shows the different parts of paint that the students mix together.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parts of Red Paint</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Parts of White Paint</td>
<td>3</td>
<td>4</td>
<td>8</td>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

Two mixtures of paint will be the same shade if the red paint and the white paint are in the same ratio. How many different shades of paint did the students make? Explain.
Talk About It

Solve the problems below as a group.

7 Refer to the problem in Question 6. Which shades of paint are the most red? Why?

________________________________________________________________________

________________________________________________________________________

8 Use the table in Question 6. Plot a point for each ordered pair. After you plot each point, draw a line connecting the point to (0, 0).

---

9 Based on the graph, what do the mixtures that are the same shade have in common? What does this tell you about their relationship?

________________________________________________________________________

________________________________________________________________________

Try It Another Way

Work with your group to determine whether the equation represents a proportional relationship. Explain your choice. You may want to make a table or a graph on separate paper to support your reasoning.

10 \( y = 2x + 4 \) ________________________________

11 \( y = 2x \) ________________________________
Talk through these problems as a class. Then write your answers below.

**Compare:** The graphs below show the number of points you earn in each level of a game. Which games, if any, have a proportional relationship between the number of points you earn and the level of the game? In which game can you earn the most points in Level 2? Explain your answer.

![Game A Graph](image)
![Game B Graph](image)
![Game C Graph](image)

---

**Apply:** Servers at a snack shop use the table below to find the total cost for frozen yogurt, but some of the numbers have worn off. If the total cost is proportional to the number of cups of frozen yogurt, find the missing numbers in the table.

<table>
<thead>
<tr>
<th>Number of Cups of Frozen Yogurt</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost ($)</td>
<td>18.00</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Analyze:** Michael says that the difference between Dani’s and Raj’s ages is always the same, so Raj’s age is proportional to Dani’s age. Is Michael correct? Explain.

<table>
<thead>
<tr>
<th></th>
<th>2010</th>
<th>2015</th>
<th>2020</th>
<th>2025</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dani’s Age</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>Raj’s Age</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
</tr>
</tbody>
</table>
Paige works in an art store that sells square pieces of canvas. There are 5 different squares to choose from.

<table>
<thead>
<tr>
<th>Canvas</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of side (in feet)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

A. Make a table to show the perimeter for each square piece of canvas. Use the formula $P = 4s$. Then draw a graph to compare the length of a side of each square to its perimeter. Use your table and graph to explain whether this is a proportional relationship.

B. Make a table to show the area for each square piece of canvas. Use the equation $A = s^2$. Then draw a graph to compare the length of a side of each square to its area. Use your table and graph to explain whether this is a proportional relationship.
Lesson 10  (Student Book pages 88–93)

Understand Proportional Relationships

LESSON OBJECTIVES

• Determine whether two quantities are in a proportional relationship from looking at quantities in a table, lines in the coordinate plane, and equations. (Use equivalent fraction relationships and multiplication/division to find proportional ratios.)

• Identify the constant of proportionality (unit rate) in a table and represented by an equation.

PREREQUISITE SKILLS

In order to be proficient with the concepts in this lesson, students should:

• Understand ratio, unit rate, and proportions.

• Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios or equations.

• Graph ordered pairs from a table on a coordinate grid.

• Recognize and generate simple equivalent fractions, including writing whole numbers as fractions.

VOCABULARY

proportional relationship: the relationship among a group of ratios that are equivalent

constant of proportionality: what the unit rate is called in a proportional relationship

THE LEARNING PROGRESSION

The ability to represent a relationship in multiple ways—through words, equations, tables of values, or graphs—and to move smoothly among them gives students a range of tools to identify the relationships and solve problems involving them.

Students have worked with proportional relationships using tables and equivalent ratios. In this lesson, they learn that the graph of a proportional relationship is a straight line that passes through the origin. They learn that another name for the unit rate is the constant of proportionality. They use these concepts to analyze relationships that may or may not be proportional.

They write equations to describe proportional relationships in the form of \( y = mx \), in which \( m \) is the constant of proportionality. Working with different methods aids in flexible thinking. Students can apply their understanding to solve a range of problems in school and everyday life. In later lessons and grades, they will connect proportional relationships to linear and non-linear functions.

CCLS Focus

7.RP.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

STANDARDS FOR MATHEMATICAL PRACTICE: SMP 3, 4 (see page A9 for full text)
Part 1: Introduction

AT A GLANCE

Students review the idea that data displayed in a table show a proportional relationship if all the ratios formed are equivalent. They learn that the ratio expressed as the unit rate is called the constant of proportionality.

STEP BY STEP

• Introduce the Question at the top of the page.
• Reinforce the definitions of proportional relationship and constant of proportionality. Have a volunteer explain what a unit rate is and relate it to the constant of proportionality.
• Read the first part of Think with students. Make sure students can connect the data in the first table with the ratios and the equations. Relate all the representations to the context of movie tickets.
• Read the second part of Think with students. Ask how the simplified ratios formed by the data in the second table are different from those formed by the data in the first. Emphasize that when the ratios are not equivalent, the data do not show a proportional relationship.

Concept Extension

Reinforce the connection between constant of variation and unit rate.

Materials: dictionary

• Write constant of variation on the board. Say that variation means change.
• Have students look up the word constant in the dictionary. Have them read the various definitions and decide which definition best applies to the term constant of variation.
• Have volunteers describe the meaning of constant of variation in their own words.
• Have students explain why a unit rate expresses a constant rate of change and therefore can be called the constant of variation.

Mathematical Discourse

• Relationships can be described in equations and in words. The relationship of total cost to tickets is shown in the equation \( c = 8t \). How could you describe the relationship in a “word equation”?
  Responses should convey the idea that the total cost of the tickets is 8 times the number of tickets.
• How would your word equation be different if the situation were about teams and players?
  The total number of players is 8 times the number of teams.
• Think of something in our class or school that \( c = 8t \) could describe and use it in a word equation.
  Students might suggest desks in a group, students at lunch tables, or weeks in a semester.
**AT A GLANCE**

Students explore how to use graphs to determine whether or not relationships are proportional.

**STEP BY STEP**

- Read Think with the students. Ask students how they can represent the data in a table using a graph.
- Have students compare and contrast the two graphs. Discuss why the first graph shows a proportional relationship but the second graph does not.
- After students have read the information in the table, have them restate each statement in their own words.
- Have students read and reply to the Reflect directive.

**ELL Support**

- Sketch examples and non-examples of *straight line* and *through the origin* on the board. Model the correct language such as, *This line goes through the origin but it is not a straight line* or *This is a straight line that does not go through the origin*.
- Have a volunteer go to the board and draw an example or non-example on a coordinate plane. The volunteer will call on classmates to describe the graph using *straight line* and *through the origin*. Repeat with other volunteers.
- Once students are comfortable with the vocabulary, tie the terms to the graphs of proportional and non-proportional relationships.

**Mathematical Discourse**

Extend the discussion of the Reflect directive with these questions.

- **Can you repeat that method in your own words?**
  Responses should paraphrase how the student found the constant of proportionality from the graph.
- **Is there another way to find the constant of proportionality?**
  Responses could include making a table of ratios from the points on the line, using the y-coordinate of the point where $x = 1$, or recognizing that each point is 8 units higher on the y-axis.
Students examine data in tables to see if they represent proportional relationships.

**STEP BY STEP**

- Tell students that they will have time to work individually on the Explore It problems on this page and then share their responses in groups. You may choose to work through the first problem together as a class.

- As students work individually, circulate among them. This is an opportunity to assess student understanding and address student misconceptions. Use the Mathematical Discourse questions to engage student thinking.

- For the second table, suggest to students that they can use either equivalent ratios or graphs to determine if the relationships are proportional.

- Help students understand what they are being asked to find in the last problem. Help them connect their answer to the idea of equivalent ratios.

- Take note of students who are still having difficulty and wait to see if their understanding progresses as they work in their groups during the next part of the lesson.

**STUDENT MISCONCEPTION ALERT:** Some students may find the ratios but not remember that all the ratios must be the same for the data to be proportional and have a constant of proportionality. Have students find and simplify the ratios for each problem. Then note that there can be only one constant of proportionality. If the simplified ratios are not equivalent, ask students why they cannot pick one of them to be the constant of proportionality. Then reinforce the idea that the relationship is not proportional.

---

**Mathematical Discourse**

- **How can you tell if the data in the table form equivalent ratios?**

  Responses might indicate that they all simplify to the same ratio.

- **Do you think you should check every ratio before you decide if the relationship is proportional or not? Why or why not?**

  Responses might include that you can recognize a non-proportional relationship with the first non-equivalent ratio.

- **If the relationship is proportional, how do you find the constant of proportionality? Could you do it another way?**

  Responses might use the term “unit rate” or indicate that it is the ratio with the denominator of 1.
Part 2: Guided Instruction

Lesson 10

AT A GLANCE

Students graph data from a table to see if there is a proportional relationship.

STEP BY STEP

• Organize students into pairs or groups. You may choose to work through the first Talk About It problem together as a class.

• Walk around to each group, listen to, and join in on discussions at different points. Use the Mathematical Discourse questions to help support or extend students’ thinking.

• If students need more support, have them use the Hands-On-Activity to help them visualize the common ratios.

• Direct the group’s attention to Try It Another Way. Have a volunteer from each group come to the board to present a table or graph that illustrates the group’s solutions to problems 10 and 11.

Hands-On Activity

Use concrete materials to model ratios.

Materials: red paper, white paper, scissors, drawing paper, glue sticks

• Have students cut 12 small squares from red paper and 30 from white paper.

• Have them divide a sheet of drawing paper into 5 sections.

• Students should use glue and the small squares to illustrate the following ratios of red paint to white paint: 1 to 3, 2 to 4, 4 to 8, 2 to 6, and 3 to 9.

• Direct students to write 2 or 3 sentences that identify the two sets of equivalent ratios and explain why they are equivalent.

Mathematical Discourse

• For the Try It Another Way problem, what did your group do to get started with the questions?

  Responses may include making a table of values and graphing or checking equivalent ratios.

• Did other groups use a different way to decide which relationship is proportional?

  Listen for responses that show students have connected the form of the equation to proportional and non-proportional relationships and encourage explanation as a preview to upcoming lessons.

• How can you use your method to decide if $y = 3x + 6$ is proportional?

  Responses should indicate understanding of the method.
Part 3: Guided Practice

Lesson 10

AT A GLANCE

Students demonstrate their understanding of proportional relationships as they examine relationships represented using both graphs and tables.

STEP BY STEP

- Discuss each Connect It problem as a class using the discussion points outlined below.

Compare:

- As students evaluate each graph, have them identify the two features that show whether or not a graph shows a proportional relationship.
- Use the following to lead a class discussion that relates the idea of a constant ratio to the graphs:
  
  What is the number of points possible for Level 1 of each game? [A: 50; B: 100; C: 100]

  Do you think the ratio of points per level will remain constant for all the levels of each game? [Only for Games A and C]

Apply:

- The second problem focuses on the idea of the unit rate or constant of proportionality.
- Once students find the unit rate, have them explain how they can use it to find the total cost of other amounts of yogurt.

Analyze:

- This problem requires students to focus on the multiplicative aspects of proportional relationships.
- Have students suggest different ways to show whether or not the data is proportional. If they use ratios, discuss why there is no constant of proportionality. If students chose to use a graph, have them explain why it does not display a proportional relationship.

- After students have determined that the data are not proportional, have them examine the table. Ask questions such as:

  How do the boys’ ages compare when you go from one column to the next? [Raj’s age is always 5 more than Dani’s.]

  Do you create a ratio table for a proportional table by adding? [No]

  How does this confirm that the data are not proportional? [They are obtained by adding, not multiplying.]

SMP Tip: Encourage students to support their answers by referring to the characteristics of the graph or the idea of equivalent common ratios. This helps them practice constructing viable arguments and critique the reasoning of others (SMP 3) as they explain whether or not the relationships are proportional.

Games A and C have a proportional relationship. The points are on straight lines that go through the origin; you earn more points in Game C. (The constant of proportionality is 100.)

No, Michael is not correct. The difference between their ages is always the same, but none of the ratios \( \frac{5}{5} \), \( \frac{10}{10} \), \( \frac{15}{15} \), or \( \frac{20}{20} \) are equivalent, so the ages are not in a proportional relationship.
AT A GLANCE

Students generate one table of data that compares side length and perimeter and another that compares side length and area. They analyze the data using both ratios and graphs to determine if the data are proportional.

STEP BY STEP

• Direct students to complete the Put It Together task on their own.

• As students work on their own, walk around to assess their progress and understanding, to answer their questions, and to give additional support, if needed.

• If time permits, have students share their tables and graphs and explain why they do or do not show a proportional relationship.

SCORING RUBRICS

See student facsimile page for possible student answers.

<table>
<thead>
<tr>
<th>Points</th>
<th>Expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>The response demonstrates the student’s mathematical understanding of how to show that a relationship is proportional using both • a table of equivalent ratios and • a graph of a straight line passing through the origin.</td>
</tr>
<tr>
<td>1</td>
<td>The student was able to show that the data are proportional using either a table of equivalent ratios or a graph, but not both.</td>
</tr>
<tr>
<td>0</td>
<td>There is no response or the response does not demonstrate that the data are proportional.</td>
</tr>
</tbody>
</table>

A

<table>
<thead>
<tr>
<th>Points</th>
<th>Expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>The response demonstrates the student’s mathematical understanding of how to show that a relationship is not proportional because • the ratios formed by the data in the table are not equivalent and • the graph formed by the data is not a straight line.</td>
</tr>
<tr>
<td>1</td>
<td>The student was able to show that the data are not proportional by showing that the ratios formed are not equivalent or the graph formed is not a straight line, but not both.</td>
</tr>
<tr>
<td>0</td>
<td>There is no response or the response does not demonstrate that the data are not proportional.</td>
</tr>
</tbody>
</table>

B
Differentiated Instruction

**Intervention Activity**

**Use graphs to model proportional and non-proportional relationships.**

**Materials:** graph paper

Students will connect graphs, ratios, and proportional relationships.

Students should label the left half of a sheet of graph paper “Proportional” and draw and label a coordinate plane. They should plot 5 points that are part of a line passing through the origin. Beneath the graph, have them record the data in a table with rows labeled x and y. Have them find and simplify the ratios, x:y. Review the idea of the constant of proportionality and have them record their constant of proportionality below the table.

Have students label the right half “Not Proportional” and repeat the process with 5 points that are not part of a straight line passing through the origin. After they find and simplify the ratios, x:y, discuss why the data do not have a constant of proportionality.

**On-Level Activity**

**Analyze real-world situations to see if they are proportional.**

**Materials:** graph paper

Students will generate data from real-world situations and then analyze the relationships to see if they are proportional.

Write the following information on the board.

- **Video Plan A:** $2 for each video you rent
- **Video Plan B:** $1 for each video you rent plus a $10 monthly fee

Have students make a table of data for each plan to show the amount it would cost to rent various numbers of videos in one month. After they have generated the data, ask students to describe two methods they can use to tell whether or not either plan represents a proportional relationship. Then have them work in pairs to analyze each set of data using both ratios and a graph. They should then explain why Plan A is a proportional relationship and name the constant of proportionality.

**Challenge Activity**

**Develop and interpret a proportional relationship.**

**Materials:** graph paper

Students will develop and interpret a proportional relationship from a point on a coordinate plane.

Have students plot one point such as (3, 6) or (5, 2) on a coordinate plane. They should connect the origin and their point and extend the line to the edge of the paper. Have them identify several other points on the line and enter the coordinates in a table with rows labeled x and y.

Have students work individually to find the following:

- the ratio of x to y in simplest form for each point
- the constant of proportionality
- an equation that relates x and y
- a real-world situation that could be modeled by their data

Have students share their work in small groups. They should explain how the graph, the table, the equation, and the real-world situation are related.